# Longman Exam Series HKDSE Mathematics Mock Exam Papers (Compulsory)

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# **Solution Guide CD-ROM Included**



# PEARSON

#### ALWAYS LEARNING

#### **Compulsory Part**

# Question Style in HKDSE Paper 1

Candidates should pay close attention to the following question types in the examination.

## Short questions in Section B

There may be some short questions in Section B which carry less marks. Since such questions may not have parts to guide candidates, they are set to differentiate high ability candidates from others. Candidates who aim to obtain 5\*\* in the HKDSE should strive to master this type of question.

#### Examples in this book:



## **Questions on real-life situations**

These questions require candidates to apply mathematical knowledge and techniques to real-life situations. Some of them may relate to the topics on '**Further Applications**' in the syllabus.

#### Examples in this book:



# Question Style in HKDSE Paper 2

According to the latest HKDSE papers, <u>more questions are set using variables and notations rather than</u> <u>numerical values</u>. Candidates should prepare to handle such questions.

### **About Number and Algebra**

#### Examples in this book:

D)



## About Measures, Shape and Space

#### Examples in this book:



## **About Data Handling**

#### Examples in this book:



#### **SECTION B (35 marks)**

Answers written in the margins will not be marked.

15. The graph in Figure 6 shows the linear relation between  $\log_9 x$  and  $\log_{27} y$ . The slope and the intercept on the horizontal axis of the graph are  $\frac{1}{4}$  and -4 respectively. Express the relation between *x* and *y* in the form  $y = Ax^k$ , where *A* and *k* are constants. (3 marks)



Figure 6

Answers written in the margins will not be marked.

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Page total

14. The following stem-and-leaf diagram shows the distribution of the ages of the members of a tennis club.

Stem (tens)	Le	eaf (	uni	ts)			
1	7	8	8	9	9		
2	0	0	1	3	3	<u>b</u>	9
3	1	2	3	5	9	9	
4	0	8	8	<u>a</u>			

It is known that the median and the range of the distribution are 27 and 31 respectively.

(a) Find a and b.

Answers written in the margins will not be marked.

- (b) A few days later, both the oldest and the youngest members leave the club, while Dennis and his father join the club. It is found that the changes in members have not changed the mode and the median of the distribution.
  - (i) Find the age of Dennis's father.
  - (ii) Is it possible that Dennis is 25 years old? Explain your answer.
  - (iii) The ages of the members of a badminton club are shown as follow:

17, 22, 22, 32, 35, 36, 39, 40

One member from the badminton club and one member from the tennis club are randomly selected to be the club representatives. Find the probability that the sum of the ages of the two representatives exceeds 65.

(6 marks)

(3 marks)

Answers written in the margins will not be marked.



- **28.** A box contains seven balls marked with the numbers -3, -2, -1, 1, 2, 3 and 4 respectively. If two balls are drawn randomly from the box at the same time, find the probability that the product of the numbers on the balls drawn is positive.
  - A.  $\frac{5}{42}$ B.  $\frac{1}{6}$ C.  $\frac{2}{7}$ D.  $\frac{3}{7}$
- **29.** The stem-and-leaf diagram below shows the distribution of the numbers of books read by a group of students in the first school term.

Stem (tens)	Lea	af (ur	nits)								
0	7										
1	3	8	9								
2	0	h	4	5	6	8	8				
3	0	0	0	0	1	k	7	7	7	8	9

If the inter-quartile range of the above distribution is at least 14, which of the following must be true?

I. 
$$0 \le h \le 4$$

II. 
$$4 \le k \le 7$$

- III.  $1 \le k h \le 3$ 
  - A. I only
  - B. II only
  - C. I and III only
  - D. II and III only

- **21.** In the figure, O is the centre of the circle ABCD. If  $\angle AOD = 120^{\circ}$  and  $\angle ODC = 70^{\circ}$ , find  $\angle ABC$ .
  - A. 100°
  - B. 110°
  - C. 120°
  - D. 140°



- **22.** In the figure, *O* is the centre of the circle *ABCDEF*.  $\triangle PQR$  intersects the circle at *A*, *B*, *C*, *D*, *E* and *F*. If  $\angle POQ = 118^{\circ}$  and AB = CD = EF, then  $\angle PRQ =$ 
  - A. 28°.
  - B. 56°.
  - C. 59°.
  - D. 62°.



- **23.** If an exterior angle of a regular *n*-sided polygon is smaller than an interior angle by  $90^{\circ}$ , which of the following is/are true?
  - I. The value of n is 8.
  - II. The number of the diagonals of the polygon is 8.
  - III. The number of axes of reflectional symmetry of the polygon is 8.
    - A. I only
    - B. II only
    - C. I and III only
    - D. II and III only

# Longman Exam Series HKDSE Mathematics Multiple Choice Questions

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# **Solution Guide CD-ROM Included**

- HKDSE and HKCEE trend analysis
- Useful techniques in tackling MCQs
- Intensive Revision Notes
- Quick Drill for checking basic skills
  - Sectional Exercises and Quizzes
  - Mock Tests (Paper 2)



# PEARSON

# **Useful Techniques in Tackling MCO**

In *HKDSE Mathematics Paper 2*, candidates need to answer <u>45 multiple choices questions within</u> <u>1 hour 15 minutes</u>. On average, candidates can only spend about **1.5 minutes on each question**. Apart from direct computation and deductive reasoning, the following six common techniques may help candidates to tackle some of the multiple choice questions quickly.

- (1) Checking for Questions WITHOUT Graphs
- (2) Checking for Questions WITH Graphs
- (3) Observing a Particular Case
- (4) Substituting a Particular Value
- (5) Elimination
- (6) Construction

# **Technique 1:** Checking for Questions WITHOUT Graphs

## Example

If the 2nd term and the 5th term of an arithmetic sequence are 13 and 1 respectively, find the 4th term of the sequence.

**A.** 17

- **B.** 9
- **C.** 5
- **D.** −4

(Reference: HKDSE Sample Paper 2009 Q36)

# Solution

#### (by Checking)

The arithmetic sequence is:

\_\_\_\_\_, 13 , \_\_\_\_\_, \_\_\_\_, 1

Option C: The 4th term is 5. Then, the common difference = 1 - 5 = -4

The following arithmetic sequence obtained satisfies all the given conditions:

17,13,9,5,1

 $\therefore$  The answer is C.

#### Strategy: by Checking

Consider the options one by one and check which of them satisfies all the given conditions.

#### (by Direct Computation)

Let *a* and *d* be the first term and the common difference of the sequence.

```
∴ 2nd term = 13

∴ a + d = 13 .....(1)

∴ 5th term = 1

∴ a + 4d = 1 .....(2)

(2) - (1): 3d = -12

d = -4

From (1), a + d = 13

(a + d) + 2d = 13 + 2d

a + 3d = 13 + 2(-4)

= 5
```

```
\therefore The 4th term is 5.
```

```
\therefore The answer is C.
```

# 8.3 Quadratic Equations

## A Solving quadratic equations by the factor method

If (ax + b)(cx - d) = 0, where  $a, c \neq 0$ , then

$$x = -\frac{b}{a}$$
 or  $x = \frac{d}{c}$ .

#### **B** Forming quadratic equations with given real roots

If  $\alpha$  and  $\beta$  are the **roots** of a quadratic equation in *x*, then the equation can be written as:

$$(x - \alpha)(x - \beta) = 0$$

🕵 Common Mistakes 📰 🖉 🖉

Students may mistakenly write  $(x + \alpha)(x + \beta) = 0$  when forming a quadratic equation with two given roots  $\alpha$  and  $\beta$ .

## C Solving quadratic equations by the quadratic formula

If  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ᠫ Exam Tips 🔜

Some MC questions require students to find roots of a quadratic equation. If the given options involve surds, that means the equation may not be easily solved by the factor method. In such case, we should solve the equation using the quadratic formula.

#### D Solving quadratic equations by the graphical method



If the *x*-intercepts of the graph of  $y = ax^2 + bx + c$  are *r* and *s*, then the roots of the quadratic equation  $ax^2 + bx + c = 0$  are also *r* and *s*.

#### 💫 Common Mistakes 📰 🔳 🚽

Some careless students may think a = r and b = s. In fact, the values of a and b cannot be found directly by reading the graph of  $y = ax^2 + bx + c$ .

Quick Drill 3 If 4(x + 5)(2x - 7) = 0, then x =A. 5 or  $\frac{2}{7}$ . B. 4 or  $\frac{7}{2}$ . C. -5 or  $\frac{7}{2}$ . D. 4 or  $\frac{2}{7}$ .

#### • Quick Drill 4

Form a quadratic equation in x whose roots are -4 and  $\frac{2}{5}$ . A. (x - 4)(5x + 2) = 0B. (x + 4)(5x - 2) = 0C. (x - 4)(2x + 5) = 0D. (x + 4)(2x - 5) = 0

# Quick Drill 5 Solve $x^2 + 3x - 1 = 0$ . A. $x = \frac{-3 \pm \sqrt{5}}{2}$ B. $x = \frac{3 \pm \sqrt{5}}{2}$ C. $x = \frac{-3 \pm \sqrt{13}}{2}$ D. $x = \frac{3 \pm \sqrt{13}}{2}$

#### • Quick Drill 6

Which of the following equations may be represented by the given graph?



# HROSE AND HRCEE MATHEMATICS PAPER 2 QUESTIONS DISTRIBUTION

		Past Public Exam Questions
17.1	Chords of a Circle	<b>10</b> , Q49
Α	Perpendiculars to chords	
В	Distances between chords and centre	
17.2	Angles in a Circle	<b>05</b> , Q25; <b>06</b> , Q46; <b>09</b> , Q48, Q49;
Α	Angles at the centre and angles at the circumference	Sample Paper, Q21
В	Angles in the same segment	
17.3	Relationships among Arcs, Chords and Angles	<b>01</b> , Q32; <b>02</b> , Q29; <b>03</b> , Q25; <b>04</b> , Q50; <b>05</b> , Q51
Α	Equal arcs, equal chords and equal angles	
В	Arcs proportional to angles at the centre	
17.4	Cyclic Quadrilaterals	<b>03</b> , Q50, Q51; <b>07</b> , Q48; <b>08</b> , Q50;
		Sample Paper, Q22
17.5	Tests for Concyclic Points	

# EXERCISE

# Part I Sectional Exercise

## 17.1 Chords of a Circle

- 1. In the figure, *AMB* and *BNC* are straight lines. AB = 10 cm and BC = 16 cm. Find the area of rectangle *OMBN*.
  - **A.**  $30 \text{ cm}^2$
  - **B.**  $40 \text{ cm}^2$
  - **C.**  $80 \text{ cm}^2$
  - **D.**  $160 \text{ cm}^2$



- 2. In the figure, *AMB* and *COM* are straight lines. If AM = MB = 8 cm and OM = 6 cm, find *CM*.
  - **A.** 10 cm
  - **B.** 12 cm
  - **C.** 15 cm
  - **D.** 16 cm



3. In the figure, *AOB* is a diameter of the circle and *DMC* is a straight line. If AB = 12 cm and DC = 10 cm, then OM =



- 4. In the figure, *AMB* and *OMC* are straight lines. If AM = MB = 12 cm and OC = 15 cm, find *MC*.
  - **A.** 6 cm
  - **B.** 7.5 cm
  - **C.** 9 cm
  - **D.** 10.5 cm



C

**40.** The figure shows  $\triangle ABC$ . *FD* intersects *BE* at G. If  $FD \perp BC$  and  $BE \perp AC$ , which of the following is/are true?



- A, F, G and E are concyclic. I.
- II. *E*, *G*, *D* and *C* are concyclic.
- III. *A*, *E*, *D* and *B* are concyclic.
- A. I only
- B. II only
- С. I and III only
- **D.** II and III only

#### **Miscellaneous Questions** Part II

- 41. In the figure, the radius of the circle is 7. AKB and *CKD* are straight lines. If AB = 12 and CD = 10, then OK =
  - A.  $\sqrt{32}$ .

B.

C.



- **42.** In the figure, *OC* intersects the chord *AB* at *M*. AM = MB = 3 and MC = 2. Find OM.
  - 1.25 Α.



С. 3





- **43.** In the figure, *ABC* is a straight line. If OB = 3and OA = 4, find BC.
  - Α. 1.4 B. 1.6 С. 1.8
  - **D.** 2



0

Α

- **44.** In the figure,  $\angle AOB = \angle BOC$  and OA // CB. Which of the following is/are true?
  - I. AB = BC
  - II. BC = OB
  - III. OC // AB
  - A. I only
  - **B.** II only
  - C. I and II only



- **45.** In the figure, *M* and *N* are the mid-points of the chords AB and CD respectively. ONM is a straight line. If AB : CD = 1 : 2, which of the following is/are true?
  - I. AB:CD = 1:2М II. ON: OM = 1:2D III.  $\cos x : \cos y = 1 : 2$ OA. I only III only **B**. C. I and III only
  - **D.** II and III only



**60.** In the figure, the two circles intersect at *G* and *D*. *ACE* is a triangle. Which of the following is/are true?



- I. *BCEF* is a cyclic quadrilateral.
- II. *ABGF* is a cyclic quadrilateral.
- III.  $\triangle ABF \sim \triangle AEC$
- A. I only B. II only
- C. I and III only D. I, II and III

## Quiz



1. In the figure, AB = 20 and OA = 26. Find the shortest distance from *O* to chord *AB*.



- 2. The figure shows two concentric circles with common centre *O*. *ABMCD* is a straight line with BM = MC = 6. OA = 12 and OC = 10. Find *AD*.
  - A.  $8\sqrt{5}$
  - **B.**  $10\sqrt{5}$
  - **C.**  $5\sqrt{10}$



**D.**  $6\sqrt{10}$ 



- **3.** In the figure, *OA* // *CB*. Find  $\angle BAC$ .
  - A. 15°B. 20°
  - **C.** 35°
  - **D.** 75°



- **4.** In the figure, *AB* is a diameter of the circle. *DEB* and *CEO* are straight lines. Find *x*.
  - **A.** 25°
  - **B.** 26°
  - **C.** 27°
  - **D.** 28°



There are 30 questions in Section A and 15 questions in Section B. The diagrams in this paper are not necessarily drawn to scale. Choose the best answer for each question.

## **Section A**

**1.** 
$$(-2^{222})^3 \left(\frac{1}{2}\right)^{666} =$$
  
**A.** -1  
**B.** 0  
**C.** 1  
**D.**  $\frac{1}{2^{441}}$ 

2. If 
$$3x - 4y = 5xy$$
, then  $y = 3x$ 

A. 
$$\frac{3x}{5x+4}$$
. B.  $\frac{3x}{5x-4}$   
C.  $\frac{5x+4}{3x}$ . D.  $\frac{5x-4}{3x}$ 

- 3. ab bc ad + cd =A. (a - c)(d - b) B. (a - c)(b - d)C. (a + c)(d - b) D. (a + c)(b - d)
- **4.** Which of the following is an identity/are identities?
  - I.  $4x^2 9 = 0$ II.  $4x^2 - 9 = (2x - 3)^2$ III.  $4x^2 - 9 = (2x - 3)(2x + 3)$ A. II only B. III only
  - C. I and II only D. II and III only
- 5. Let  $f(x) = -2x^2 + x 1$ . If f(a) = f(-a), find the value(s) of *a*.
  - **A.** 0 **B.** 1 **C.** 0 or  $\frac{1}{2}$  **D.** 0 or  $-\frac{1}{2}$
- 6. Let  $f(x) = -2x^3 + x^2 x + k$ . If x + 1 is a factor of f(x), find the remainder when f(x) is divided by x 2.

<b>A</b> .	-18	В.	-12
C.	18	D.	24

356

- 7. Which of the following about the graph of quadratic function  $y = -1 2(x 3)^2$  are true?
  - I. The graph has no *x*-intercepts.
  - II. The coordinates of the vertex are (-3, -1).
  - III. The axis of symmetry of the graph is x 3 = 0.
  - **A.** I and II only **B.** I and III only
  - C. II and III only D. I, II and III
- 8. Find the range of values of k such that the quadratic equation  $x^2 4x + k = 2$  has real roots.
  - A.  $k \le 4$  B.  $k \ge 4$  

     C.  $k \le 6$  D.  $k \ge 6$
- **9.** If *a* and *b* are real numbers such that *ab* < 0, which of the following must be true?
  - I.  $\frac{a}{b} < 0$ II. a + b < 0III.  $a^2 - b^2 < 0$ A. I only B. II only C. I and III only
  - D. II and III only
- 10. If x is a positive integer satisfying the inequality  $3x^2 10x 8 < 0$ , then the smallest possible value of x is

А.	0.	B.	1.
C.	3.	D.	4.

## Section **B**

31. 
$$\frac{x+2}{x^2-5x+6} - \frac{x-3}{x^2-x-2} =$$
  
A. 
$$\frac{3x+11}{(x-3)(x-2)(x+1)}$$
  
B. 
$$\frac{3x-7}{(x-3)(x-2)(x+1)}$$
  
C. 
$$\frac{-3x+11}{(x-3)(x-2)(x+1)}$$
  
D. 
$$\frac{9x-7}{(x-3)(x-2)(x+1)}$$

**32.** The graph in the figure shows the linear relation between *x* and  $\log_4 y$ . If  $y = ab^x$ , find the values of *a* and *b*.



A. 
$$a = 1, b = 2$$
  
B.  $a = 1, b = -2$   
C.  $a = 4, b = -2$ 

- **D.**  $a = 4, b = \frac{1}{2}$
- **33.** Which of the following have the same value as  $2012_{(10)}$ ?
  - I.  $2 \times 10^4 + 0 \times 10^3 + 1 \times 10^2 + 2 \times 10^1$
  - II. 7DC<sub>(16)</sub>
  - III. 11111011100<sub>(2)</sub>
  - A. I and II only
  - B. I and III only
  - C. II and III only
  - **D.** I, II and III

- **34.**  $i^{2012} =$  **A.** i **B.** -i**D.** -1
- 35. Which region in the figure represents the



- A. Region I
- B. Region II
- C. Region III
- D. Region IV
- **36.** Let *a*, *b* and *c* be positive integers. If *a*, *b*, *c* is an arithmetic sequence, which of the following must be true?
  - I.  $a^2$ , ab, ac is an arithmetic sequence.
  - II.  $2^a$ ,  $2^b$ ,  $2^c$  is a geometric sequence.
  - III.  $a^2$ ,  $b^2$ ,  $c^2$  is not an arithmetic sequence.
  - A. I and II only
  - B. I and III only
  - C. II and III only
  - **D.** I, II and III
- 37. The sum of all the negative terms in the geometric sequence  $-\frac{2}{3}, \frac{1}{2}, -\frac{3}{8}, \dots$  is

A. 
$$-\frac{8}{21}$$
 B.  $-\frac{32}{21}$ 

 C.  $-\frac{8}{3}$ 
 D.  $-\frac{32}{75}$